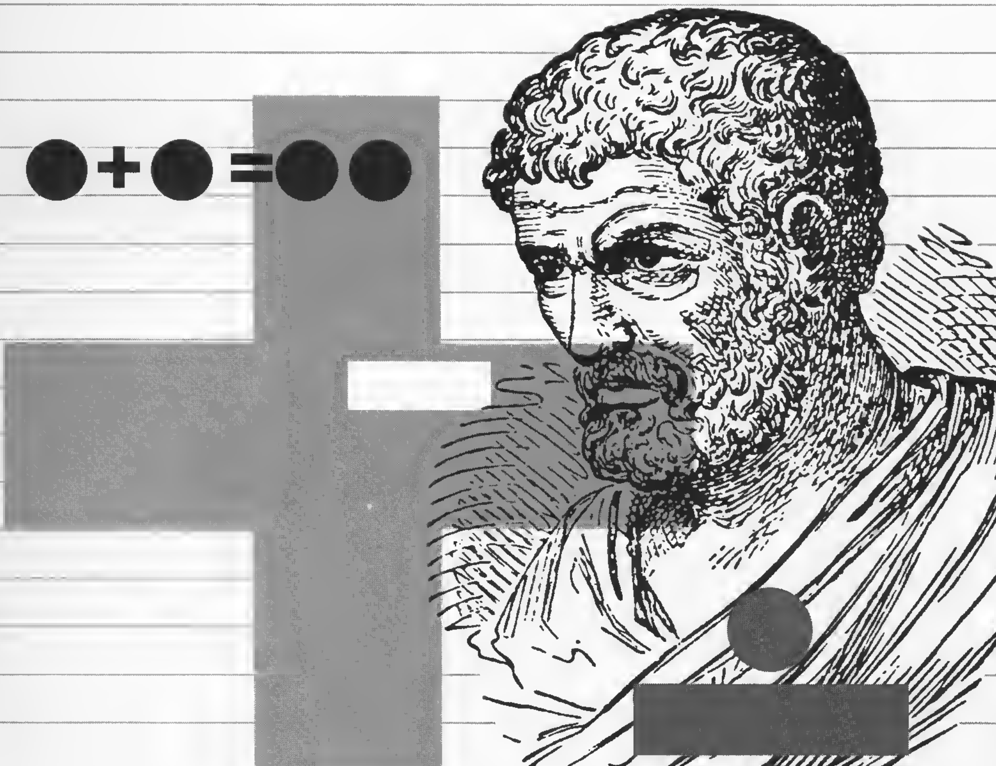
Graphing



ATARI®
LEARNING SYSTEMS

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Using This Program at Home

Many ATARI® Learning Systems program manuals were originally designed for use by teachers in the classroom. The programs themselves, however, are no less engaging and instructive for "independent learners"—children, students, and adults—working at home.

Every manual includes a "Getting Started" section that explains how to load the program into your computer system quickly and easily. Since many basic prompts and other instructions are displayed right on your screen, that's all you'll need to begin learning and exploring with most ATARI Learning Systems programs. But whether you're a parent, a tutor, or a home learner teaching yourself, it's a good idea to look through the teaching materials in your manual. You're likely to find important details on using the program, valuable supplementary information on its subject matter, and some creative ideas for getting the most educational and entertainment value out of your ATARI Learning Systems program.

Introduction

The *Graphing* module consists of six computer programs and this guide, which is designed to assist teachers in using the module in the classroom. Objectives, background information, suggestions for instructional uses, student handouts, and sample computer runs are included.

Slope, Polygraph, and Polar are problem-solving programs suitable for either individual use or large group demonstrations. By displaying on the screen the graph of the equation entered, these programs enable students to investigate relationships between equations and their graphs without tedious point-by-point plotting.

Snark is an educational game involving coordinate points and circles. It can be used by individual students or small groups to develop strategies for locating the Snark while practicing constructing circles with given centers and radii.

Introduction

Radar and ICBM are simulations of the interaction of two missiles — an ICBM and a SAM. In Radar, the positions of the missiles are shown on a simulated radar screen; in ICBM, the missiles' headings are given as north and east coordinates. Working individually or in small groups, students use a variety of techniques to determine a heading for the SAM, enter it into the computer, observe the results of their decision, and then correct the heading if necessary.

These programs are designed for use by students in grades 7 through 12 in a variety of precalculus mathematics classes.

Handout pages in this guide may be duplicated for use with students.

Index to Programs on Diskette

Slope

Graphs up to five linear equations on one coordinate system

Polygraph

Graphs polynomials, other functions, and conics on a Cartesian coordinate system

Polar

Graphs polar equations using a polar coordinate system

Snark

Presents concepts of coordinate points and circles in a game format in which students try to locate a "snark" hidden on a grid

Radar

Simulates the interception of a missile using a radar screen display; students use angles of 0 to 359 degrees to direct their missiles

ICBM

Simulates the interception of a missile; students use graphing, ratios, slope, and trigonometric functions to determine missile headings

Getting Started

Follow these steps to load the Graphing program into your ATARI computer system:

1. With your computer turned off, turn on your television set or monitor and disk drive. Wait for the busy light on the disk drive to go out.

2. If your computer is *not* equipped with built-in ATARI BASIC, insert an ATARI BASIC cartridge in the cartridge slot (the left cartridge slot on the ATARI 800® computer).

3. Insert the Graphing diskette in your disk drive (disk drive 1, if you have more than one drive) and close the disk drive door or latch.

4. Turn on your computer. As your disk drive goes to work, you'll hear a beeping sound while the first part of the program loads into your computer. After several moments, a title screen will appear on your screen, followed by a menu of program selections.

Because your computer loads portions of the program as you use them, you must leave the Graphing diskette in your disk drive while using the program.

Some questions asked by the Graphing program require a simple Yes or No answer. You may respond by typing **YES** or **NO**, or simply by typing **Y** or **N**. Always press **RETURN** to confirm your response to a question. You may usually change your response before pressing **RETURN**; just use the **DELETE BACK SPACE** key to delete your original response, then type in the new response.

To return to the program menu, hold down the **ESC** key. When the question Do you want to try again? appears, type **N** and press **RETURN**.

Slope

Graphing Linear Equations

Specific Topic:	Graphing, analytic geometry
Type:	Problem-solving
Reading Level:	8 (Fry)
Grade Level:	8 – 11

Description

Slope graphs linear equations of the form $y = mx + b$ or $x = b$. Each equation is entered in algebraic form with no extra computation symbols.

Objectives

- To observe that ordered pairs (x,y) satisfy equations of the form $y = mx + b$ where m and b are constants for one equation
- To observe that equations of the form $y = mx + b$ are straight lines when graphed
- To discover the relationship between m in the equation of the form $y = mx + b$ and the slope of the line when graphed
- To relate the b in the equation $y = mx + b$ with the y-coordinate where the line crosses the y-axis (y-intercept)

Slope

- To identify through graphing the common properties that exist in a family of equations, such as parallel and perpendicular lines
- To identify examples of parallel, perpendicular, horizontal, and vertical lines

Background Information

Slope allows the student to plot up to five linear equations on one coordinate system. All equations must be in the form $y = mx + b$ or $x = b$.

For example:
$$y = 2x + 5$$
$$y = \tfrac{1}{2}x - 3$$
$$y = .5x + 1$$
$$y = 3$$
$$y = -2$$

Equations that will *not* be accepted are these:

$y = x^2 + 2$ (all must be first degree)
$r = 5s + 5$ (variables must be x and y)

Since the program restricts the range of the y axis to -4 to $+4$, it is suggested that the y intercept be not greater than 4 or less than -4. The x axis extends from -8 to 8.

If lines requiring a larger grid or second degree equations are to be plotted, use the program Polygraph, which is also included in this module.

The graphs of the equations are color coded, so the use of a color television is highly recommended.

Slope

Use in an Instructional Setting

Preparation

Introduce the concepts of plotting points on a graph. This could be accomplished through the Hurkle program from *Metric and Problem Solving*. Option 4 of Hurkle uses a coordinate system from -5 to 5.

Students should also be able to substitute values for x and determine the y coordinate to get ordered pairs that satisfy the equation. The ordered pair should be graphed, and the students should realize that $y = mx + b$ forms a straight line. Students are then ready to work with the concepts of slope and y-intercepts.

Using the Program

In a directed lab situation, use Handouts 1, 2, and 4 for students to explore slope and intercepts.

Follow-up

You can build an entire unit of study using this program. Following the study of slope/y-intercept and parallel/perpendicular relations, investigations on triangles, quadrilaterals, and distance concepts can introduce new topics or present new insights. For example, have students graph the following sets of equations on one pair of axes:

Set 1	**Set 2**
$y = -3$	$y = -2/3x + 3$
$x = 5$	$y = -2/3x - 3$
$y = \frac{1}{2}x + 3$	$y = 3/2x + 3$
	$y = 3/2x - 3$

Answer these questions for the above sets of equations:

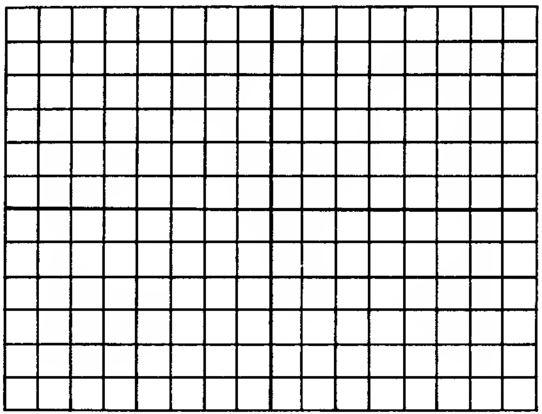
What figure is bounded by these lines?
How can you informally "prove" it?
What is the area enclosed by these lines?

Investigating Slope

Name ______________________________

Class ________________ Date ______________

1. Using the Slope program, graph the following equations. Copy the graphs and label each graph on the grid.



a. $y = 1x + 2$

b. $y = 2x + 2$

c. $y = 3x + 2$

d. $y = \frac{1}{2}x + 2$

e. $y = \frac{1}{4}x + 2$

The equations are of the form $y = mx + b$, where "m" and "b" are numbers.

As you look at the graphs of these equations, what seems to happen to the graph as “m” gets larger?

What seems to happen as “m” gets smaller?

What does “m” seem to do in the equation?

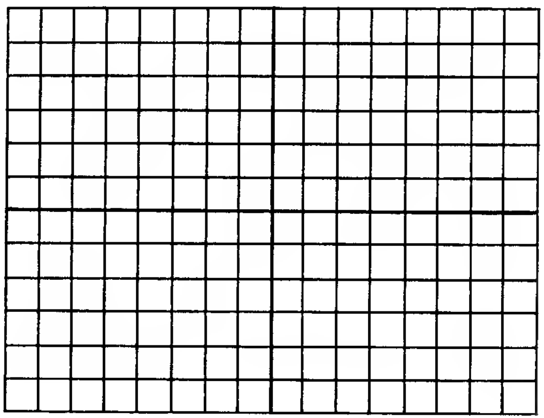
Try some more equations to verify your answers. Record the equations you tried.

Investigating Slope (Page 2)

Name

Class Date

2. Using the Slope program, graph the following equations. Draw the graphs and label each graph on the grid.



a. $y = 2x + 1$

b. $y = -\frac{1}{2}x + 1$

c. $y = -3x + 1$

d. $y = x + 1$

e. $y = -\frac{1}{4}x + 1$

As you look at the graphs, what effect does the negative "m" value have?

What conclusion can you draw from this?

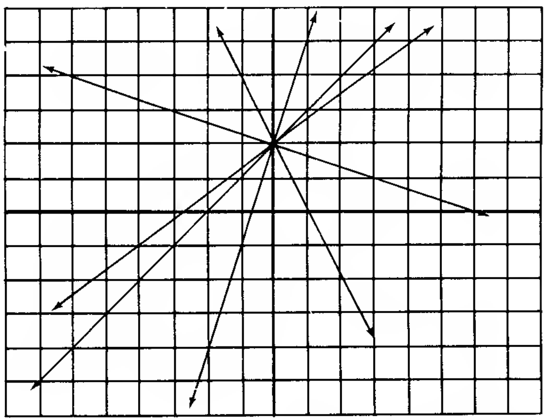
Try some more equations to verify your conclusion. Record the equations you tried.

Investigating Slope (Page 3)

Name ______________________

Class ______________ Date ______________

3. Match the equations with the graphs. Put the correct letter on each graph. Use the Slope program to check your answers.



a. $y = 3x + 2$

b. $y = x + 2$

c. $y = -\frac{1}{3}x + 2$

d. $y = \frac{3}{4}x + 2$

e. $y = -2x + 2$

What point is common to all the graphs? (,)

What number is the same in each of the equations? ____

4. Think about the two equations below:

$y = 3x + 1$

$y = 3x - 3$

Since the "m" value is the same for both, what do you think the graphs would look like?

____

____

Check out your ideas, using the Slope program.

Make up two equations that would have graphs that are parallel to the graph of

$y = -\frac{1}{2}x + 1$ ____________________ ____________________

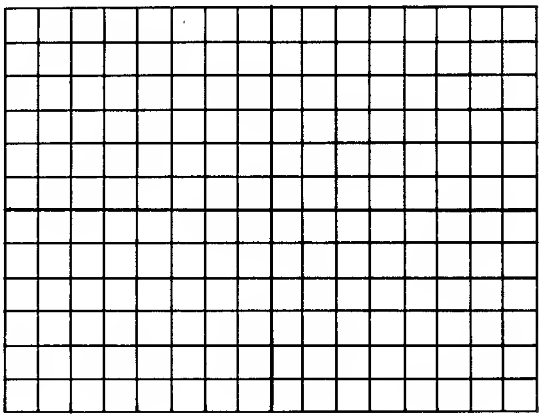
Use the Slope program to test your equations.

Investigating Intercepts

Name ______________________

Class ______________ Date ______________

Graph and label the following equations on the axis below:



a. $y = x + 2$

b. $y = x$

c. $y = x - 5$

d. $y = x + \frac{1}{2}$

e. $y = x - \frac{2}{3}$

In the equations ($y = mx + b$), the "m" values are all 1, but the values for "b" vary. What happens as "b" gets larger? ______

What is the role of "b" in $y = mx + b$? ______

Linear Equations

Name ______________________________

Class __________________ Date __________

Use Slope to investigate the role of "m" and "b" in $y = mx + b$.

Approach this task any way you wish, but keep notes on what you do. Record the equations you use.

1. What is the role "m" in the equation $y = mx + b$? ______________________

How did you discover this?

Linear Equations (Page 2)

Name ______________________________

Class ________________ Date ____________

2. What is the role of "b" in $y = mx + b$? ______________________

____

____

What did you do to discover this? ______________________

____

____

3. Write the equations for two lines that would be parallel to:

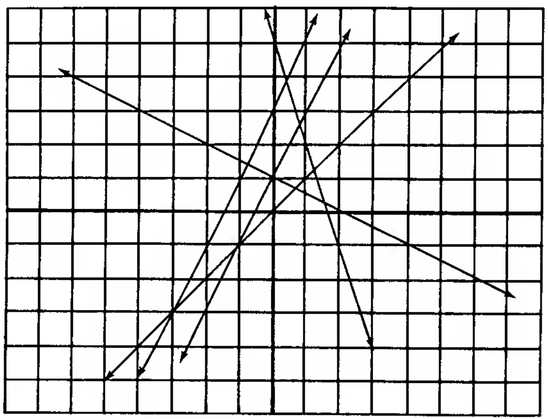
$y = \frac{1}{2}x + 3$

____

____

Verify your choices by using the Slope program. Are the graphs of your equations parallel to $y = \frac{1}{2}x + 3$?

4. Match the following equations and graphs by writing the letter of the equation next to its graph. Use Slope to verify your answers.



a. $y = x$

b. $y = 2x + 3$

c. $y = -3x + 5$

d. $y = 1 - \frac{1}{2}x$

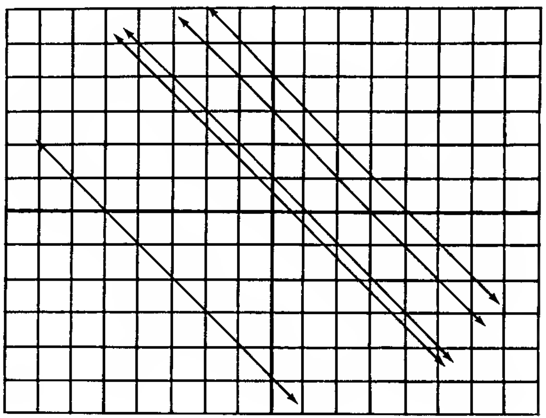
e. $y = 2x + 1$

Linear Equations (Page 3)

Name ______________________________

Class ________________ Date ______________

5. Match the graphs with the equations. Verify your answers, using Slope.



a. $y = -x + 3$

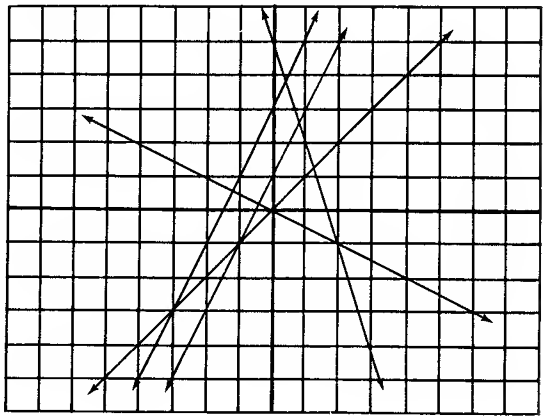
b. $y = -x + 4$

c. $y = -x - 5$

d. $y = -x + \frac{1}{2}$

e. $y = -x + 1$

6. Match the following equations and graphs. Use Slope to verify your answers.



a. $y = x$

b. $y = 2x + 3$

c. $y = -3x + 5$

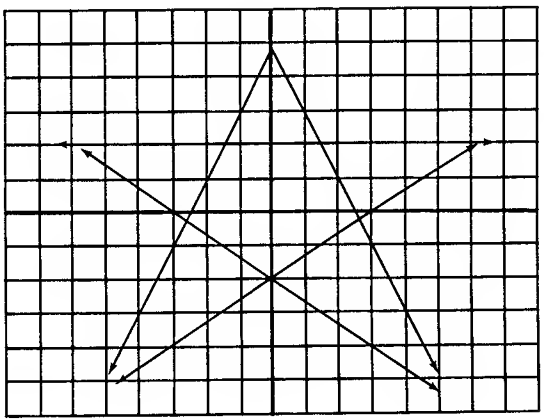
d. $y = -\frac{1}{2}x$

e. $y = 2x + 1$

Equations from Graphs

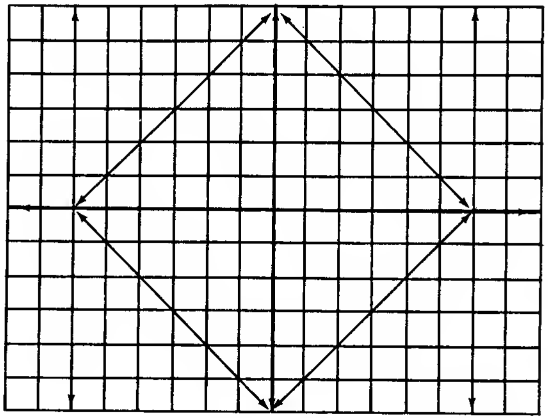
Name ______________________

Class ____________ Date ____________



Write equations that will make this figure.

1. ______________________
2. ______________________
3. ______________________
4. ______________________
5. ______________________



Verify your equations with Slope.

Write equations that will make this figure.

1. ______________________

2. ______________________

3. ______________________

4. ______________________

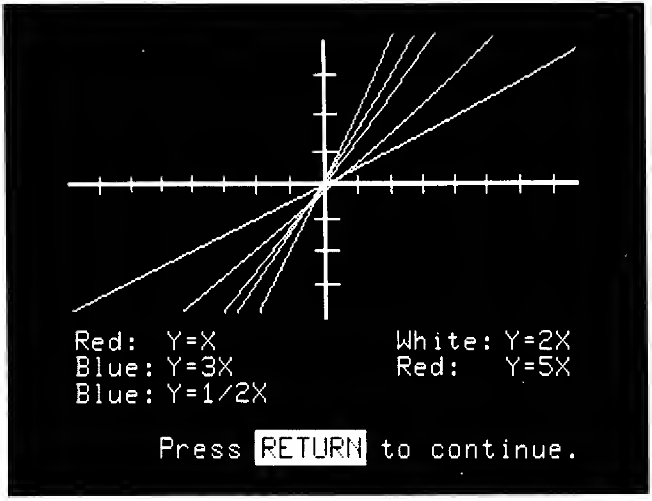
5. ______________________

6. ______________________

Sample Runs

After all of the following equations are entered, the screen at the left results.

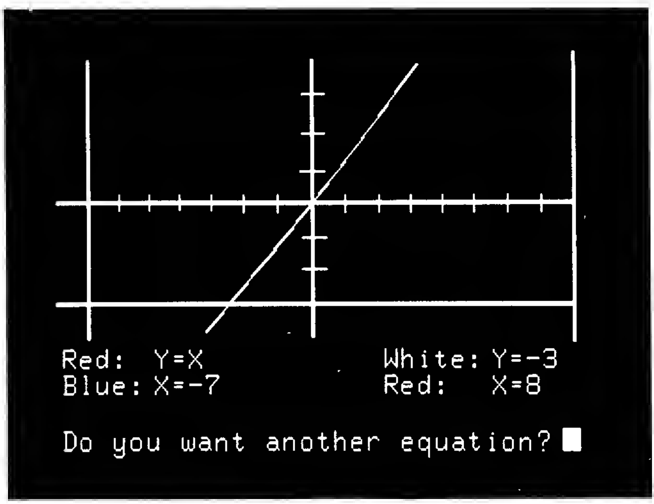
y = x
y = 2x
y = 3x
y = 5x
y = ½x



Five equations are the most the program will graph on one pair of axes.

The screen at the left resulted after the following equations were entered:

y = x
y = −3
x = −7
x = 8



Examples of Screen Output

Graphing on a Rectangular Coordinate System

Specific Topic:	Algebra, senior mathematics
Type:	Problem Solving
Reading Level:	9 (Fog)
Grade Level:	9 – Higher education

Description

Polygraph allows the plotting of most relations in the x-y plane. After the relation is plotted, the student can view various sections of the coordinate plane or can “zoom in” on the graph. Several functions may be plotted and compared on the same set of axes.

Objectives

- To develop skills in graphing by identifying axes, scale, origin, coordinates, roots, slope, and intercepts
- To explore Cartesian coordinates by graphing with various scales, specifying maximums and minimums, and expanding or shrinking distances and sections of the graphs
- To discover properties of trigonometric functions: their limits, their continuity, and their periodicity

Polygraph

Background Information

- To analyze graphs of quadratics and find their roots
- To investigate the graphs of conics

In the study of analytic geometry, the ability to graphically represent equations is important. This program plots relations in Cartesian coordinates. Mathematical relationships can be investigated without the tedious work involved in plotting points by hand.

The Polygraph program is divided into three main parts:

1) Establishment of plotting parameters

2) Entry of the equation

3) Exploration of the current plot

Establishment of plotting parameters

The boundary conditions of the graph must be defined. First the coordinate axes must be described.

What is the minimum X value? Maximum X value?

Next specify the range for Y (the vertical axis). The student can enter the Y range, or the program will determine it.

Do you want the Y axis computed? YES.

The program will attempt to put as much of the plot as possible on the screen. If the program does the Y-axis scaling, the spacing can be forced to be equal with the X-axis. This allows circles, for example, to be plotted more accurately, eliminating the distortion in the X and Y directions on the screen. If the student chooses equal spacing, he or she must enter the the Y value that will appear in the middle of the screen. This is referred to as the "mean Y value." Usually the mean Y value will be zero (0).

Do you want equal spacing on the Y axis?
YES.
What is the mean Y value?

The number of plot points must be entered. Remember that the more points the smoother the curve, but the longer the time required for the plot. A smooth graph usually requires 100 to 200 points.

How many points do you want plotted? 100.

Entry of the equation

All equations entered into Polygraph must be in a form the program can read. Every equation must be of the form:

Y = expression
or
Y^2 = expression

The second form is designed for use with conics, where both the positive and negative square roots must be considered.

The expression consists of the variable "X" used in conjunction with the following operators:

+	addition
−	subtraction
*	multiplication
/	division
^	exponentiation

Parentheses may be used to override the normal hierarchy of operations. The standard hierarchy is:

^	exponentiation
*/	multiplication and division from left to right
+ −	addition and subtraction from left to right

Several built-in mathematical functions are also available. Functions are always evaluated before any operations are performed. The functions available are as follows:

SIN	sin
COS	cosine
ATN	arctangent
INT	greatest integer
RND	random number
SGN	sign
ABS	absolute value
SQR	square root
EXP	exponential
LOG	natural logarithm
CLOG	logarithm base ten

A number of other desirable functions can be derived using the above functions. (Refer to the *ATARI BASIC Reference Manual.*)

Exploration of the current plot

After the equation is entered, it will be plotted. The student can then move the screen window to explore the plot further. (The screen window is the section of the plot currently shown on the television screen.)

These are the screen window commands:

U—Move the screen window *u*p from current window.

D—Move the screen window *d*own from current window.

L—Move *l*eft of current window.

R—Move *r*ight of current window.

C—Move in *c*loser (zoom in).

F—Move *f*arther away (zoom away).

RETURN—Return to plotting parameters section.

Note that when one of these options is used, only the last equation graphed will appear on the screen. Any other graphs on the axes will be erased.

Use in an Instructional Setting

Preparation

Of basic importance to the understanding of algebra and analytic geometry is the ability to estimate and visualize the characteristics of polynomial functions. Polygraph can be a valuable tool to the student in developing these essential skills, because it allows for investigation that—were typical paper and pencil techniques for graphing to be used—would otherwise likely end in frustration.

The following list gives examples of good sample equations for Polygraph.

Y	=	2*x	(straight line)
Y	=	(X+1)^2	(parabola)
Y	=	X^2+2*X+1	(parabola)
Y^2	=	4-X^2	(circle)
Y	=	ATN(X/SQR(-X*X+1))	(inverse sine)

In the exercises on Handout 5, Investigating Quadratic Equations, Polygraph is used to sketch quadratic functions. This is only one example of Polygraph's applications. You're encouraged to incorporate Polygraph into the study of other mathematical equations. One goal might be to teach students to estimate roots of a function.

Polygraph

In the study of quadratic functions, the student should have an understanding of the following principles:

- Determining the concavity of a quadratic function, whether it is increasing or decreasing
- Determining the roots of a quadratic function by estimating the x-intercepts of the graph of the function
- Adding a constant to a function, affecting the vertical position of the function
- Relating how the coefficient of the quadratic term affects the graph of a function
- Determining what causes a function to change its horizontal position when graphed

Using the Program

The program is not intended to be a stand-alone activity in the classroom. Polygraph should be incorporated into the classroom investigations as a tool to remove much of the drudgery of graphing.

Two different examples of laboratory activities using Polygraph are illustrated in the handouts. The first — on Handout 5, Investigating Quadratic Equations — is used to study the following forms of quadratic equations:

$y = ax^2$
$y = a(x-h)^2$
$y = a(x-h)^2 + k$

Before using this program, the students should have plotted ordered pairs and done some graphing of simple quadratic equations in the form $y = ax^2 + bx + c$.

In the second investigation, Handout 6, Coefficients, Constants, and Roots, students should have a good understanding of how to graph quadratic functions, along with the concepts of root, concavity, coefficient, and the standard form of the quadratic, $ax^2 + bx + c$. The purpose of the activity is to discover a method for predicting the roots of the equation.

Follow-up

The following are other subject areas and some possible topics for using Polygraph.

Algebra
properties of polynomials
simultaneous linear equations
points and distances in the plane
plotting equations
factoring

Intermediate Algebra
Cartesian studies
graphs of functions
symmetry
slope and intercepts of linear functions
quadratic functions
conics
logarithmic functions

Analytic Geometry (Introductory Analysis)
relations and functions
polynomial functions
tangent to a curve
limits and continuity
slopes and derivatives
inverse functions
maxima and minima
circular and trigonometric functions

Science
exponential growth or decay
trajectories of projectiles
planetary motion
wave physics
quantum physics

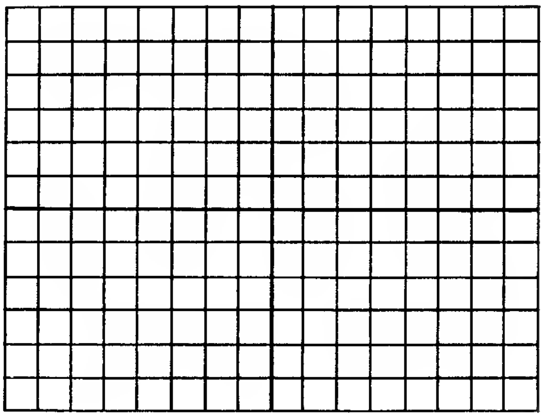
Investigating Quadratic Equations

Name ____________________

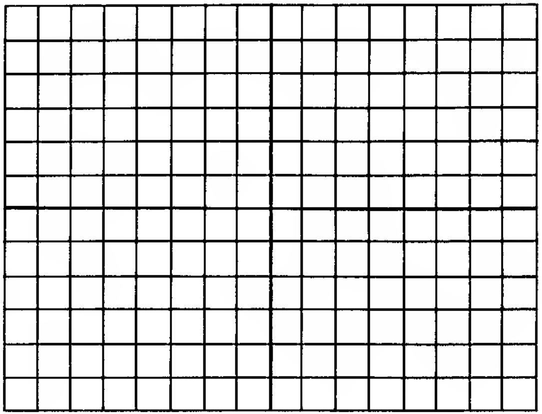
Class ____________ Date ____________

Purpose: To determine the role of ‘a’ in the equation $y = ax^2$.

1. Using Polygraph, produce graphs of the following equations. (Sketch and label these graphs on the axis provided.)



a. $y = x^2$
   $y = \frac{7}{4}x^2$
   $y = \frac{1}{4}x^2$



b. $y = -x^2$
$y = -\frac{7}{4}x^2$
$y = -\frac{1}{4}x^2$

Investigating Quadratic Equations (Page 2)

Name ______________________________

Class ________________ Date ________________

c. What does the graph look like if ‘a’ is positive?

d. What seems to happen if ‘a’ is negative?

e. What seems to happen as the absolute value of ‘a’ gets larger?

f. What seems to happen as the absolute value of ‘a’ gets smaller?

2. Write 'Upward' or 'Downward' to show the direction the graph opens for each function.

a) __________ $y = -4x^2$

b) __________ $y = \frac{1}{5}x^2$

c) __________ $y = .3x^2$

d) __________ $y = -\frac{3}{4}x^2$

3. Underline the formula in each pair that defines the function with the wider graph.

a. $y = 3x^2$, $y = x^2$

b. $y = -\frac{1}{5}x^2$, $y = -x^2$

c. $y = \frac{2}{3}x^2$, $y = -3x^2$

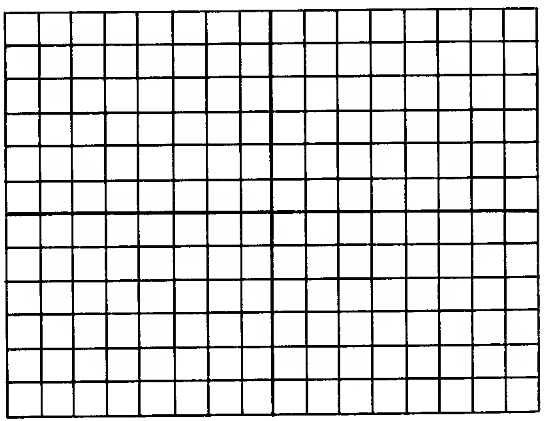
Investigating Quadratic Equations (Page 3)

Name ______________________

Class ______________ Date ______________

Purpose: To determine the role of ‘h’ in the equation $y = a(x - h)^2$

4. Use Polygraph to plot each of the following quadratic equations. Sketch and label them on the grid.



a. $y = x^2$
b. $y = (x - 5)^2$
c. $y = (x + 8)^2$

5. Study the example below. Then tell how the graph of the first function can be moved to obtain the graph of the second.
Example: The graph of $y = 3x^2$ can be moved 5 units to the *right* to obtain the graph of $y = 3(x-5)^2$.

a. The graph of $y = -2x^2$ can be moved ____ units to the ____ to obtain the graph of $y = -2(x-3)^2$.

b. The graph of $y = \frac{1}{2}x^2$ can be moved ____ units to the ____ to obtain the graph of $y = \frac{1}{2}(x+4)^2$.

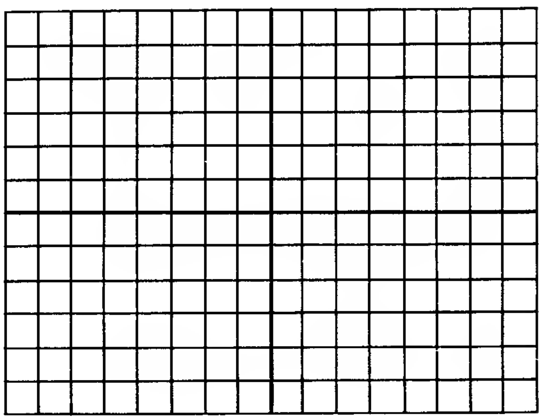
c. The graph of $y = -x^2$ can be moved ____ units to the ____ to obtain the graph of $y = -(x-10)^2$.

Investigating Quadratic Equations (Page 4)

Name ______________________________

Class ________________ Date ________________

6. Use Polygraph to plot the following equations. Sketch and label the graphs on the coordinate system provided.



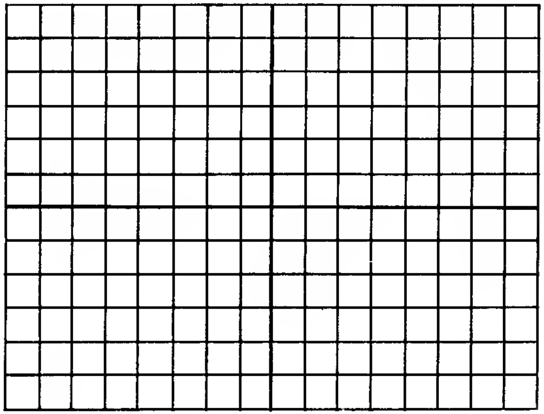
a. $y = -x^2$

b. $y = -(x-5)^2$

c. $y = -(x+6)^2$

d. $y = -(x-9)^2$

7. Obtain the graph of each of the following just as you did in Exercise 6.



a. $y = \frac{1}{2}x^2$

b. $y = \frac{1}{2}(x - 2)^2$

c. $y = \frac{1}{2}(x + 4)^2$

d. $y = \frac{1}{2}(x - 8)^2$

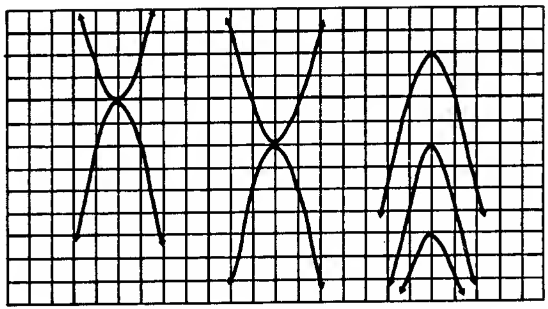
Investigating Quadratic Equations (Page 5)

Name ____________________

Class ____________ Date ____________

Purpose: To determine the role of “k” in the equation $y = a(x-h)^2 + k$

8. Match each formula for a quadratic function with one of the parabolas shown below.



a. $y = 2x^2$ Graph ________

b. $y = -2x^2$ Graph ________

c. $y = 2(x+7)^2$ Graph ________

d. $y = 2(x+7)^2 + 4$ Graph ________

e. $y = 2(x-7)^2 - 2$ Graph ________

f. $y = 2(x+7)^2 - 4$ Graph ________

g. $y = -2(x-7)^2 - 2$ Graph ________

9. Do the graphs of the functions in each pair have the same shape? Write Yes or No. Be able to explain your reasoning.

Example 1: $y = -5x^2$ and $y = -5(x+2)^2 + 7$ **Answer** Yes

Example 2: $y = 2(x+1)^2 - \frac{1}{2}$ and $y = 3x^2$ **Answer** No

a. ____________ $y = x^2 + 2$ and $y = (x-4)^2 - 3$

b. ____________ $y = (x-2)^2 + 6$ and $y = 2x^2 + 1$

c. ____________ $y = -\frac{1}{4}(x+6)^2 + 2$ and $y = -\frac{1}{4}x^2 - 5$

d. ____________ $y = 3(x+1)^2 - 4$ and $y = 3(x-8)^2 + 10$

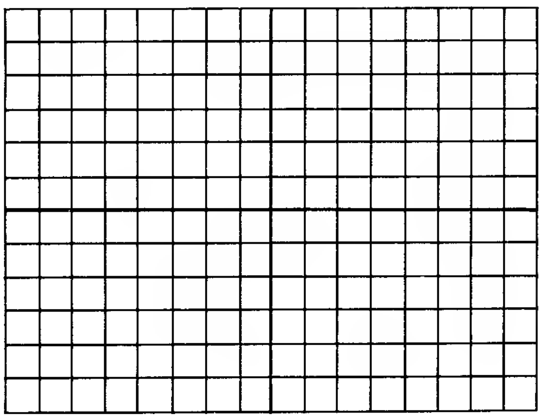
Investigating Quadratic Equations (Page 6)

Name ______________________

Class ______________ Date ______________

10. Draw the graph of $y = \frac{1}{3}x^2$ by plotting the following ordered pairs, then connecting these points with a smooth curve.

$(-3,3)$, $(-2,\frac{4}{3})$, $(-1,\frac{1}{3})$, $(0,0)$, $(1,\frac{1}{3})$, $(2,\frac{4}{3})$, $(3,3)$



Now draw the graph of each of the following functions by moving the seven points according to the value of h and k in (x,y): $y = a(x-h)^2 + k$.
Label the graphs as you sketch them.

a. $y = \frac{1}{3}(x-6)^2$

b. $y = \frac{1}{3}(x-2)^2 - 3$

c. $y = \frac{1}{3}(x+4)^2 - 2$

d. $y = \frac{1}{3}(x+6)^2 + 4$

11. Use Polygraph to verify the results of exercise 10.

12. Write a formula for the function whose graph can be obtained by each of the motions described.

Example: The graph of (x,y): $y=3x^2$
is moved 4 units to the left
and 5 units downward.

Answer:
$y=3(x+4)^2-5$

a. The graph of $y=4x^2$
is moved 1 unit to the right. ____________

b. The graph of $y=-2x^2$
is moved 3 units to the left. ____________

c. The graph of $y=-x^2$
is moved 3 units to the left
and 2 units upward. ____________

d. The graph of $y=5x^2$
is moved 2 units to the right
and 3 units downward. ____________

Coefficients, Constants, and Roots

Name ________________________________

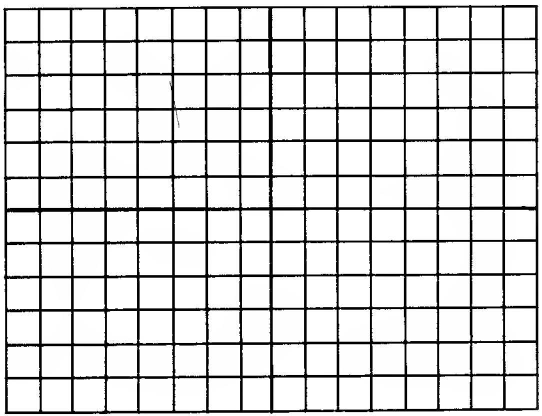
Class _______________ Date _______________

Using the Polygraph program, plot on the same set of axes the series of quadratic functions given in each problem. On the grid below, sketch what is shown on the screen.

1) Graph on the same set of axes:

a. $y = x^2$
   $y = 2x^2$
   $y = \frac{1}{2}x^2$
   $y = \frac{1}{4}x^2$

b. $y = -x^2$
   $y = -2x^2$
   $y = -\frac{1}{2}x^2$
   $y = \quad \frac{1}{4}x^2$



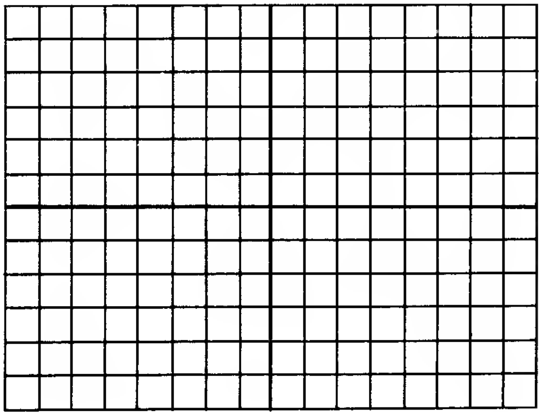
How does the coefficient of the quadratic term affect the graph?

Coefficients, Constants, and Roots (Page 2)

Name ______________________

Class ______________ Date ______________

2) Graph on the same set of axes and sketch on the grid below:



$y = x^2 - 4$

$y = x^2$

$y = \frac{1}{2}x^2 - 2$

$y = x^2 + 2$

How does adding a constant term affect the graph?

Coefficients, Constants, and Roots (Page 3)

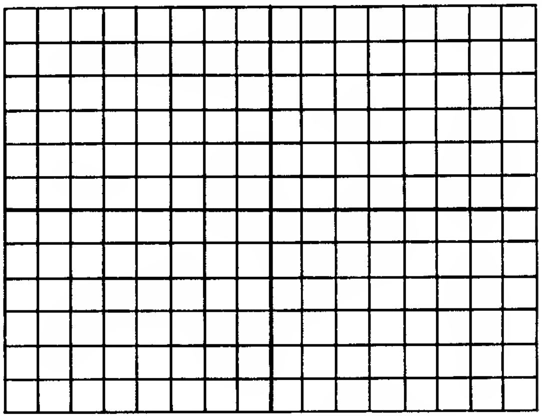
Name ______________________________

Class ____________________ Date ____________________

3) Graph on the same set of axes:

a. $y = x^2$
$y = x^2 + x$
$y = x^2 + 2x$
$y = x^2 - 2x$

b. $y = x^2 + 2x$
$y = 2x^2 + 2x$
$y = 4x^2 + 2x$



A vertical line through the vertex of a quadratic function is called the axis of symmetry. Develop a formula that will always give the equation of this line of symmetry, given the coefficients of the quadratic function $y = ax^2 + bx + c$ as a, b, and c. $x =$ ______________________

4) Factor the following equations and then plot them, using Polygraph. You may plot either the factored or unfactored form of the function. Compare your factors and the function plotted.

a. $y = x^2 - 4 = (\qquad\qquad)(\qquad\qquad)$
b. $y = x^2 + 2x + 1 = (\qquad\qquad)(\qquad\qquad)$
c. $y = x^2 + 16 = (\qquad\qquad)(\qquad\qquad)$
d. $y = 3x^2 + 12x + 8 = (\qquad\qquad)(\qquad\qquad)$

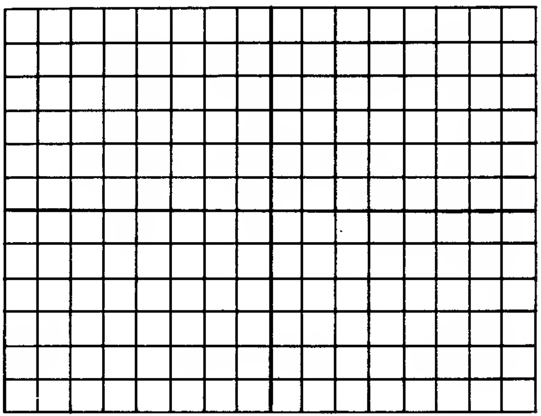
What conclusion can you draw regarding the factors and the plot of the function?

Coefficients, Constants and Roots (Page 4)

Name ______________________

Class ______________ Date ______________

6) Sketch each of the following quadratic functions. Verify your results, using Polygraph, and give estimates of the roots.



a. $y = -x^2 + 4$

b. $y = x^2 + 2x - 8$

c. $y = -\frac{1}{4}x^2 - 2x$

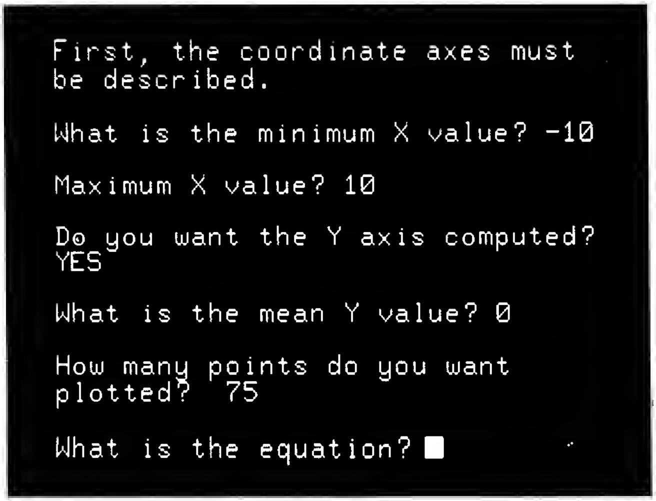
d. $y = x^2 - 6x - 16$

e. $y = x^2 - 4x$

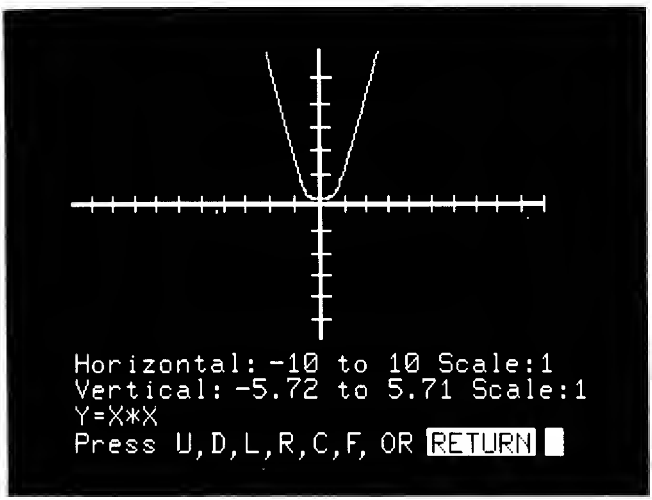
f. $y = (x + 5)^2 + 3$

Sample Runs

After the directions to the program are given, each equation is graphed with certain parameters specified by the user. After the equation is entered and **RETURN** is pressed, the graph is plotted.



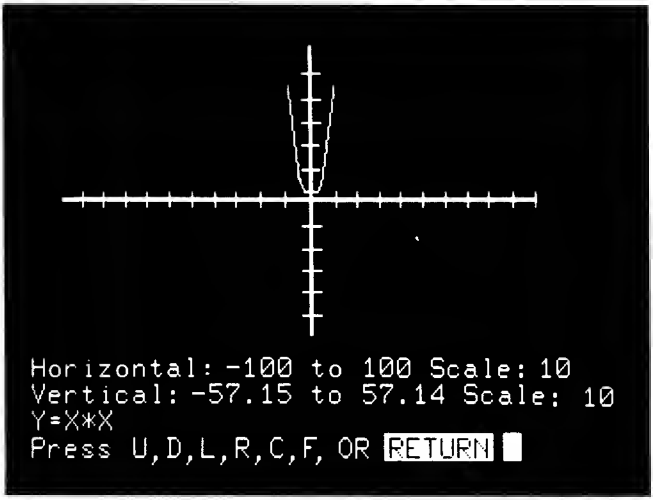
The equation is printed at the bottom of the screen, followed by the horizontal and vertical units and scale. Then the graph is plotted point-by-point. The prompt line is then printed to give options to view various parts of the graph.



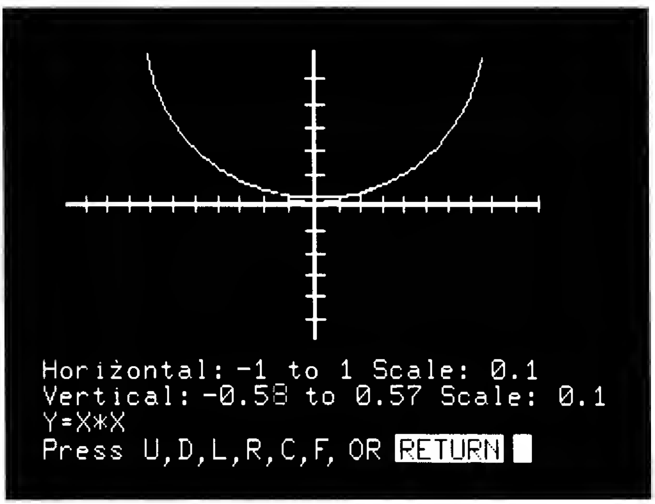
Examples of Screen Output

Sample Runs

Pressing **F** (zoom away) displays the screen shown on the left. Notice that the graph seems to lose its shape because it is being viewed from such a distance.



Pressing **C** twice (once to return to former screen and once again to shrink distance from screen) will show this screen. **C** will shrink the distance in looking at the graph or zoom in.



Polar

Graphing Equations on a Polar Coordinate System

Specific Topic:	Polar coordinates, trigonometric functions
Type:	Problem Solving
Reading Level:	8.5 (Fog)
Grade Level:	11 – Higher education

Description

Polar allows the plotting of most polar coordinate equations. After an equation is plotted, the polar plane can be varied to show various parts of the relation. The student can also zoom in to see the detail of a particular section of the relation.

Polar

Objectives

- To study the relationship between polar and rectangular coordinates through graphing of trigonometric functions
- To observe art in mathematics as it occurs in the graph of equations
- To examine trigonometric functions and analyze their equations when expressed in polar form
- To calculate distance and minimum and maximum concepts in terms of polar coordinates by using the graph of each equation
- To recognize periodicity of functions
- To distinguish between functions and relations

Polar

Background Information

In the study of polar coordinates, the ability to graph relations easily is important. Since it's difficult to achieve the accuracy needed to adequately construct a polar graph, students quickly become frustrated. Polar eliminates this frustration and allows a student to investigate a very interesting aspect of mathematics.

The Polar program is divided into three main parts:

- Establishment of plotting parameters
- Entry of the equation
- Exploration of the current plot

Establishment of plotting parameters

The program asks several questions to determine how the plot will be displayed:

Do you want the default range for theta?
Answer Yes or No. If Yes, the default range for theta is 0-360 degrees. If No, the computer will request the upper and lower limit.

Do you want the axes computed?
If yes, the program will calculate the scaling, given the radius. The program will attempt to keep the horizontal and vertical scales equal.

If no, the student will enter the distance from the pole to the edge of the screen window at 0, 90, 180, and 270 degrees. The average student will probably prefer *not* to use this option.

How many points do you want plotted? The larger the number of points, the smoother the graph will be, but the longer the plotting time will be. The number of points can be 2 to 160, but in most cases at least 50 points are needed for a clear graph.

Entry of the equation:

All equations entered into Polar must be in the following form:

R = expression
or

R^2 = expression

The second form is designed for use in situations where the complete graph requires both the positive and negative square roots.

The expression consists of the letter "O" (for theta θ) used in conjunction with these operators:

+ addition
− subtraction
* multiplication
/ division
^ exponentiation

Theta is always assumed to be in degrees, *not* radians. Parentheses may be used to override the normal hierarchy of operations. The standard hierarchy is as follows:

1.	^	Raising to a power is calculated first
2.	*/	Multiplication and division from left to right
3.	+ −	Addition and subtraction from left to right

Several mathematical functions are also available. Functions are always evaluated before any operations are performed. These are the available functions:

SIN	sine
COS	cosine
ATN	arctangent
INT	greatest integer
RND	random number
SGN	sign
ABS	absolute value
SQR	square root
EXP	exponential
LOG	natural logarithm
CLOG	logarithm base ten

These are examples of correct input into the program:

```
  R=3-SIN(0)
R^2=COS(3+0)
```

Exploration of the current plot

After the equation is entered, it will be plotted. The screen window (the section of the plot currently on the television screen) can then be moved to explore the plot further.

The screen window commands consist of the following:

U—Move the screen window *u*p from current window

D—Move the screen window *d*own from current window

L—Move *l*eft of current window

R—Move *r*ight of current window

C—Move *c*loser (zoom in)

F—Move *f*arther away (zoom away)

RETURN—Return to the establishment of plotting parameters section.

Note that when one of these options is used, only the last equation graphed will appear on the screen. Any other graphs on the axes will be erased.

Polar

Use in an Instructional Setting

Preparation

Before using Polar, students should thoroughly understand the concept of a polar coordinate system. The following polar equations are good samples to use for demonstration purposes.

r	$=$	$3 - \sin\theta$	limacon
r	$=$	$3 + \cos 5\theta$	rose curve
r	$=$	$\theta/6$	hyperbolic spiral
r^2	$=$	$\cos 2\theta$	lemniscate
r	$=$	$3/(2 + 2\cos\theta)$	parabola
r^2	$=$	$4/\theta$	lituus

Using the Program

Polar is not intended to stand alone as an activity in the classroom. It is most effectively used as part of a laboratory experience. The program should be incorporated into the classroom investigation as a tool to remove the drudgery of graphing polar equations.

Handout 7, Two Families of Polar Curves, illustrates one use of the program. The handout explores the topic of investigating the families of curves derived from plotting various forms of polar equations. It describes how the families of curves called cardioids and roses would be investigated. This might be a start of a two-day investigation of polar coordinate curve families. Other types of curves that may be investigated are spirals, lemniscates, lituus, and limacons.

Follow-up

You might include the following topics as follow-up activities. These also may use Polar.

1) Further investigations of polar curve families
2) Polar form for complex numbers
3) Transformations between Cartesian and polar coordinates
4) Polar equations of conics
5) Distance formula in polar coordinates
6) Period and amplitude of circular functions

Two Families of Polar Curves

Name ______________________________

Class ________________ Date ____________

1) Using Polar, plot the following curves. You'll need to predict the maximum value for R in each equation. Be sure to use the letter O for theta and include parentheses where shown.

   a) R = 1 + COS(O) Maximum R = ______________

   b) R = 1 − SIN(O) Maximum R = ______________

   c) R = 3*COS(O) Maximum R = ______________

   These equations are called cardioids. Make up an equation that you believe would result in a cardioid. Sketch what you believe your equation will look like. Then verify this, using Polar.

2) Using Polar, plot the following curves.

a) R=SIN(2*O)   Maximum R= ________________

b) R=SIN(3*O)   Maximum R= ________________

c) R=COS(5*O)   Maximum R= ________________

d) R=COS(4*O)   Maximum R= ________________

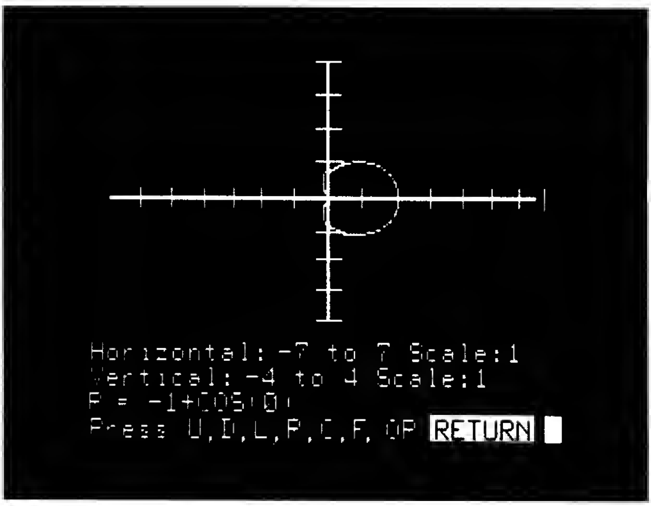
These interesting curves are called roses and are identified by the number of leaves in the curve. R=SIN(2*O), for example, is a four-leafed rose. Make up your own rose equation, decide how many leaves you think it will have, and sketch it below. Now graph your equation, using Polar. How can you identify from the equation the number of leaves a rose equation will produce?

Sample Runs

The default range for theta is 0 to 360 degrees.

Answering Yes to "Do you want the range on the axes computed?" allows the computer to calculate the best units on the axes.

The parameters at the bottom of the screen are printed after they are calculated. Then the equation is graphed point-by-point.



Examples of Screen Output

Errata Sheet

ATARI Learning Systems

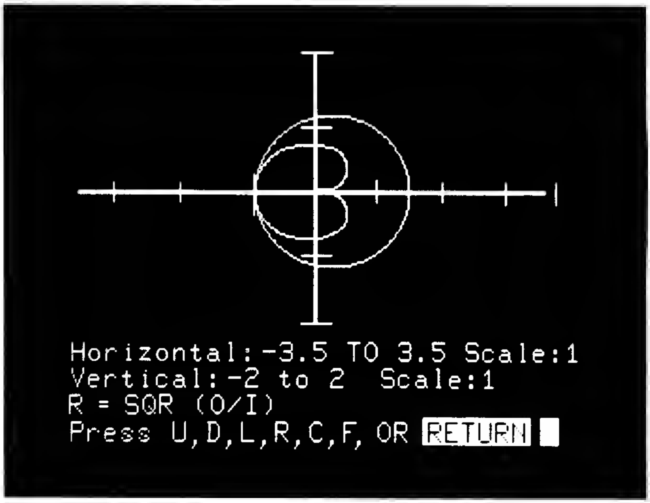
Graphing

Page 71 Both illustrations are incorrect

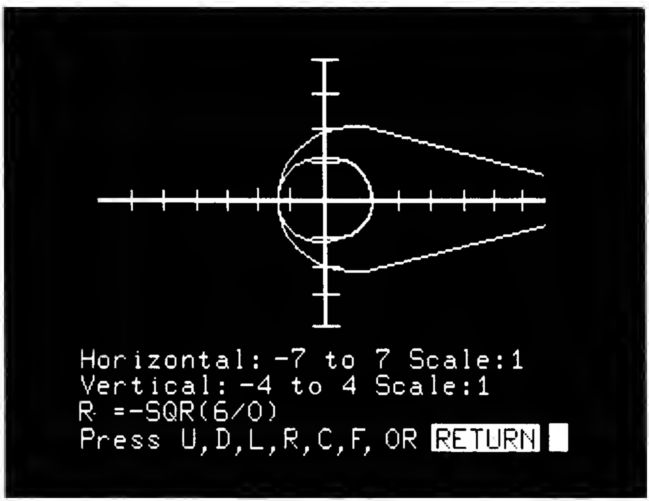
Sample Runs

This graph was drawn as a result of the following information:

Default range for theta? **YES**
Do you want the axes computed? **YES**
What is the maximum radius? **2**
How many points? **100**
What is the equation? **R^2=O/3**



This graph was drawn as a result of the following information:
Another graph? **YES**
Reuse the same equation? **NO**
Another equation on the same screen? **NO**
Default range for theta? **YES**
Do you want the axes computed? **YES**
What is the maximum radius? **4**
How many plot points? **100**
What is the equation? **R^2=6/O**



Examples of Screen Output

Snark

Coordinate System Game

Specific Topic:	Circle concepts, coordinate system
Type:	Educational Game
Reading Level:	6 (Fry)
Grade Level:	7 – 11

Description

The Snark hides on a 10×10 grid. Students must find the Snark by entering a center and the radius of a circle. The computer will tell if the Snark is inside, outside, or on the circle.

Objectives

- To identify the parts of a circle: circumference, diameter, radius, and area
- To use the Cartesian coordinate system
- To use coordinates of a point and a radius to make a circle on a grid
- To distinguish between points inside, outside, or on a circle
- To recognize when a particular strategy is appropriate, use it in selected problem situations, and determine the relevancy of data

Snark

Background Information

In Snark, students must locate a point on a grid and draw circles of a given radius. The computer "hides" the Snark at some point on a 10×10 grid. The student then selects a point that serves as the center of a circle and a radius. The computer determines whether the Snark is inside, outside, or on the circle. This continues until the student finds the Snark. The last guess will always be a point with a radius of 0, thereby locating the exact point. A compass and several copies of Handout 9 will help students locate the Snark.

Use in an Instructional Setting

Preparation

Introduce the vocabulary needed to work with a circle and grid. Identify on a circle the circumference, diameter, and radius. Discuss the region inside a circle as compared to the region outside a circle and the points on a circle. Make sure students know how to find points on a grid. Use the crossword puzzle on Handout 8, Grids and Circles, as a vocabulary exercise.

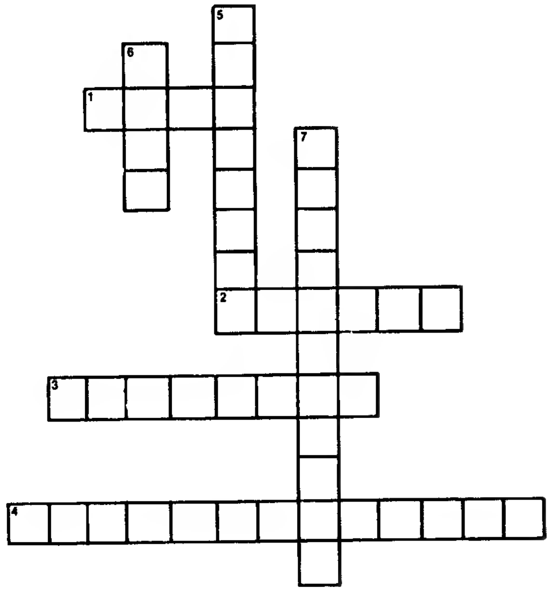
Using the Program

Use Handout 9. Practice drawing a few circles, starting from a given point and with a given radius, on one of the grids on Handout 9. Play Snark with the class. You can make a transparency from Handout 9 to assist the group in learning the game. Divide the class into groups and see which group can capture the Snark with the fewest guesses.

Grids and Circles

Name ______________________________

Class ________________ Date ________________



The Clues

Across:

1. The region inside a circle
2. The line segment from the center of the circle to the circle
3. One fourth of a coordinate grid
4. The distance around a circle

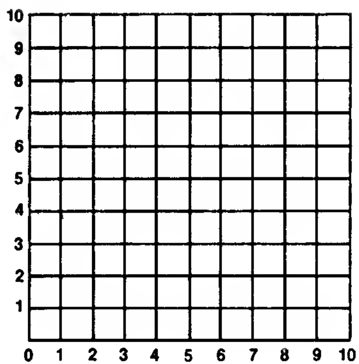
Down:

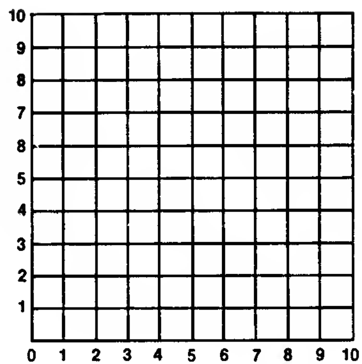
5. The line segment that passes through the center of the circle
6. An array of parallel horizontal and vertical lines
7. Two points identifying the location of a point on a grid

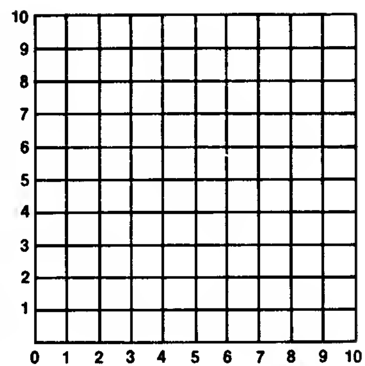
Snark

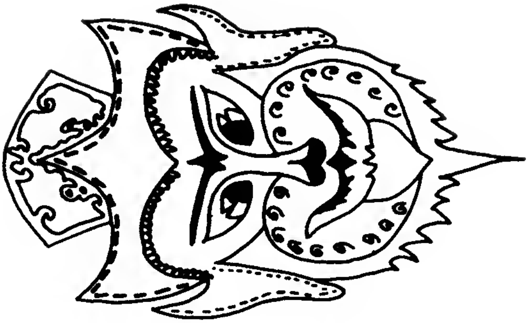
Name ______________________

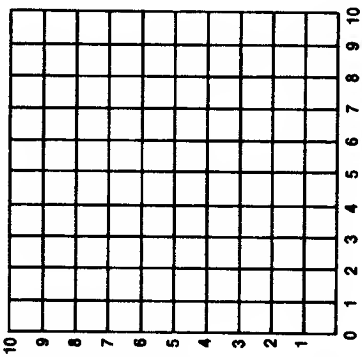
Class ______________ Date ______________

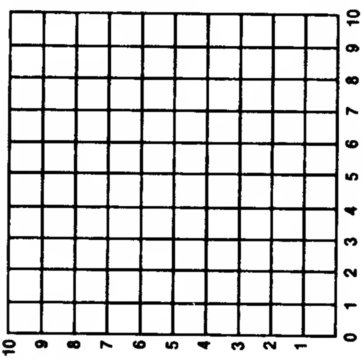




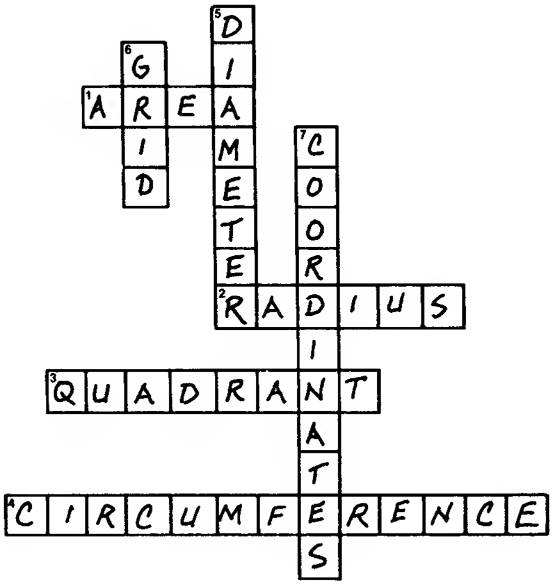





0 1 2 3 4 5 6 7 8 9 10
1 2 3 4 5 6 7 8 9 10

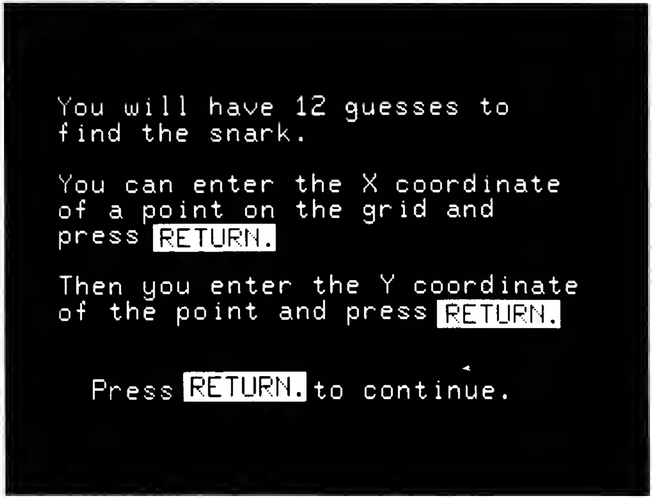

0 1 2 3 4 5 6 7 8 9 10
1 2 3 4 5 6 7 8 9 10

Answer Key

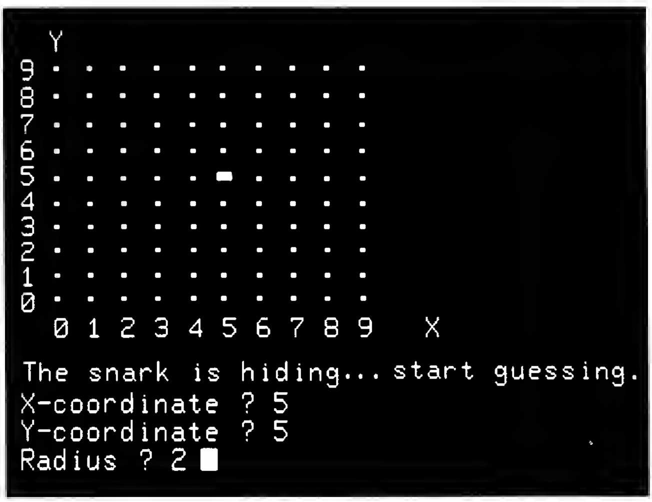

1 AREA
2 RADIUS
3 QUADRANT
4 CIRCUMFERENCE
5 DIAMETER
6 GRID
7 COORDINATES

Snark

Sample Runs



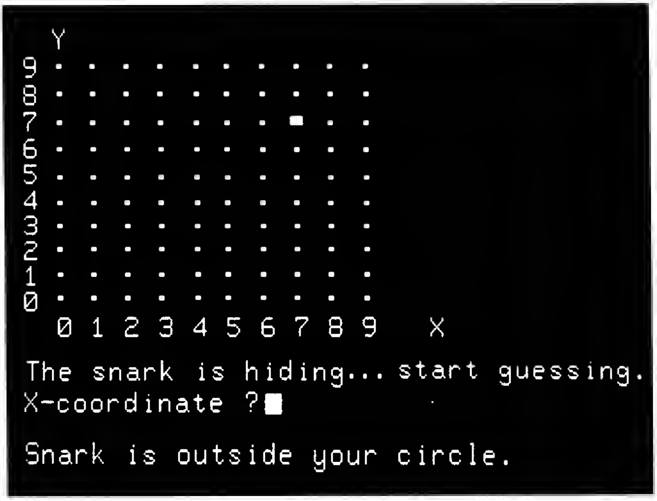
The student is given directions.



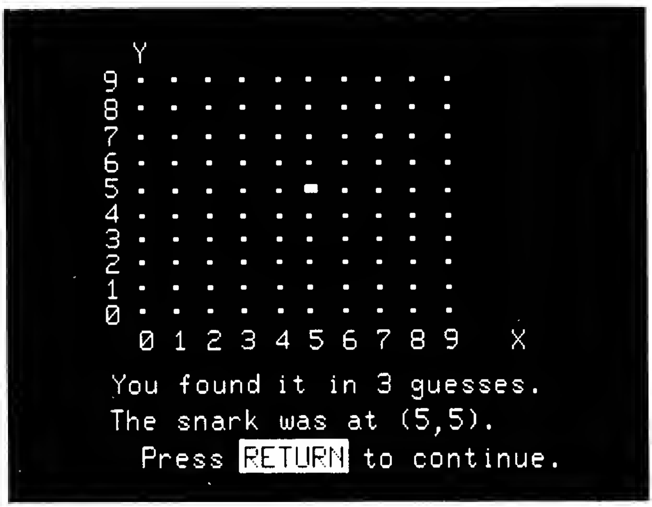
The student enters an X coordinate, a Y coordinate, and a radius.

Examples of Screen Output

Sample Runs



The computer states that the Snark is inside, outside, or on the circle.



The last guess will have a radius of zero to determine a single point.

Radar

Angle Estimation

Specific Topic:	Coordinate system, degrees of angles
Type:	Simulation
Reading Level:	4 (Fry)
Grade Level:	7 – 12

Description

Radar is a simulation in which the student guides a surface-to-air missile (SAM) toward an incoming enemy missile by specifying the SAM's heading in degrees on a coordinate system. The ICBM and the SAM appear as blips on a radar screen so the student can visually determine corrections in headings.

Objectives

- To practice estimating angles in degrees
- To use angles to make comparisons and draw conclusions

Radar

Background Information

The program simulates a radar screen on which the student guides a SAM missile toward an incoming ICBM. The student must shoot down the ICBM before it reaches its target, which is the point of origin on a coordinate system. A SAM is launched as soon as the student types in the direction for the missile. Directions are measured clockwise from north in degrees, so a heading can be any angle from 0 to 359. If the heading is an angle greater than or equal to 360°, the program subtracts a multiple of 360 from the heading to obtain an angle less than 360°. The student can change the heading at any time by typing in a new direction (angle).

The program includes the sounds of the radar blips and the simulated sound of the destruction of the ICBM or its explosion of the target. You may wish to adjust the volume of the television set.

Radar

Use in an Instructional Setting

Preparation

Students need to know that a circle can be measured in degrees. They need to be aware that in this program, zero degrees is at the top of the radar screen (north on a compass). Degrees are measured in a clockwise direction from zero.

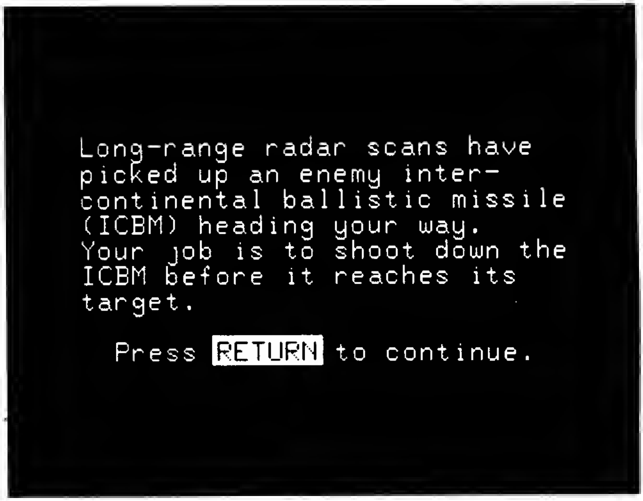
Using the Program

Radar is appropriate for individual students or small groups. It can be used to reinforce the relationship between angle size and degree measurement.

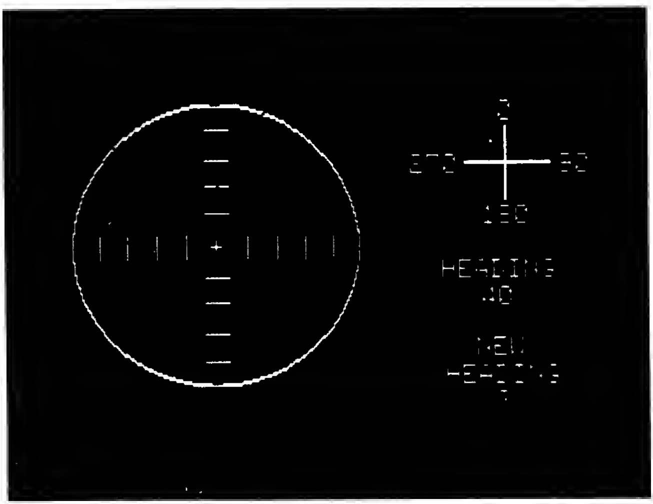
Follow-up

Radar is a simulation that demonstrates some underlying mathematical relationships important for students to understand intuitively. They can then examine these relationships more concretely by doing the ICBM program, which is included in this courseware package.

Sample Runs



Radar simulates the interception of an ICBM by a SAM missile.



Students select the heading for the SAM in degrees from 0 to 359.

Examples of Screen Output

Arc Tangent Calculation

Specific Topic:	Trigonometric functions, analytic geometry
Type:	Simulation
Reading Level:	7 (Fry)
Grade Level:	9 – 12

Description

ICBM is similar to Radar in that it simulates the interception of an enemy ICBM missile by a surface-to-air missile (SAM). However, ICBM proceeds one step further than the Radar program. In ICBM students can use a calculator or an arc tangent table to find the correct heading for their SAM missile as they attempt to seek out and destroy the incoming enemy ICBM. Unlike Radar, ICBM doesn't provide a visual representation of the missile locations, so students must determine headings for the SAM from the information on a table. The table shows north/east coordinates for both the SAM and the ICBM.

Objectives

- To use the rectangular coordinate system
- To calculate arc tangents and use an arc tangent table or calculator to find the resulting angle
- To use angles and line relationships to make comparisons and draw conclusions
- To compare two ratios and find sets of equivalent ratios
- To use appropriate strategy in selected problem situations
- To use scale drawings to determine distances, directions, locations, and the like

Background Information

In this program, the computer serves as a calculation tool and simulator. After the computer randomly launches the missile, the student has the opportunity to intercept the ICBM. However, the positions change as both the enemy ICBM and the students' intercepting missile travel through space. The computer exemplifies the real-time applications in a way that would otherwise be monumental or impossible.

ICBM is designed as a computer game for intercepting an intercontinental ballistic missile. Students do this by determining what heading the surface-to-air missile should take. With each radar scan, the student can change the course of the SAM missile. Change of direction is the only question presented. If the direction is figured within an allowable tolerance (5 miles), the surface-to-air missile closes in on the ICBM. This process continues until the ICBM is either intercepted or passed by the SAM.

Students are presented with a data table in which they give the coordinates of an enemy missile in miles north and east of the point of origin. Students must determine and enter a heading for the SAM.

ICBM		SAM		
Miles North	Miles East	Miles North	Miles East	Heading ?
886	666	0	0	

The ICBM program uses only the quadrant I portion of the coordinate system. The heading requested by the program refers to the angle created by the SAM missile from the vertical axis of the quadrant. Unlike the Radar program, the angle must fall between 0 and 90, since ICBM uses only one quadrant of the coordinate system.

If the headings are correct, the SAM destroys the ICBM. If not, the ICBM reaches its target.

Use in an Instructional Setting

Preparation

Some classwork on the chalkboard, overhead transparencies, or paper is needed to provide an appreciation of the difficulties involved in hitting a target that's not stationary. Before doing ICBM, the students should realize that heading refers to the SAM's angle of direction in degrees measured from north to east (0° to 90°).

Using the Program

You can use this program in the following ways:

- As a supplemental activity for a classroom development of introductory trigonometric functions
- As a stand-alone game for motivational purposes in studying vectors, bearings, or tangents
- As a group project before or after you introduce the concept of vectors, heading, or tangents
- As an individual exercise to increase the user's ability to solve vector and tangent problems
- As an introduction to the study of vectors with a class

The students can discover the correct heading through four different methods: 1) guessing the angle (heading); 2) calculating ratios so the ratio of north/east is the same for the ICBM as for the SAM; 3) plotting position vectors on graph paper and making direct angle measurements on the graph; 4) finding the precise, correct heading for the SAM by calculating arc tangents.

Guessing

Guessing the angle becomes a monotonous and unrewarding task. The students may start out guessing and intuitively arrive at the ratio relationship described in method 2. They might also decide they need more tools to work with, in which case they can be directed to method 3 or 4.

Ratio

Since the ICBM is headed directly toward the origin (the SAM launching site), its line of flight is completely described by its slope, Y/X (or N/E in this case). The student may attempt to intercept the ICBM by following the ICBM's line of flight in the reverse direction. Since each point on this line has the same slope — N/E — the problem is to choose successive flight headings to give the SAM the same ratio of N/E. as the ICBM.

The ICBM's ratio:

$$\frac{N\ (ICBM)}{E\ (ICBM)}$$

need be calculated only once and changed to a decimal value.

For example,

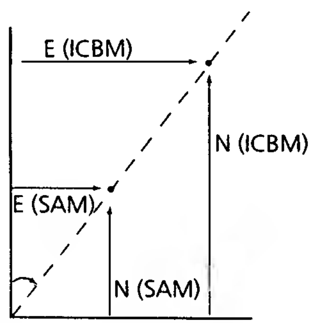
ICBM	
N	E
326	215

yields the ratio N/E = 326/215 = 1.5
The ratio for the SAM:

$$\frac{N\ (SAM)}{E\ (SAM)}$$

should be calculated for each successive position in order to determine if the next heading should be directed more to the east or the north. If the SAM ratio is greater than the ICBM ratio, then use a heading greater than the previous heading.

$$\frac{N\ (ICBM)}{E\ (ICBM)} = \frac{N\ (SAM)}{E\ (SAM)}$$



If the ratio N/E is 1, then the angle is 45°. If the N/E ratio is greater than 1, then the angle is less than 45°, and so forth. In this example, the ratio is 1.5, so an angle of 35° would be a good guess.

Graphing

Perhaps the most straightforward approach to intercepting the ICBM is to set up a graph with the SAM at the origin. The student then plots the ICBM's position and constructs a line between the two missiles. A direct angle measurement of this flight path may then be made, using a protractor. As the program progresses, successive positions of the two missiles could be plotted. This provides graphing and measuring practice.

A more challenging approach is to guess at the first few headings, then plot the succession of the previous missile positions that are displayed on the screen. Velocity vectors constructed between these points will allow more advanced students to predict the ICBM position some time in the future and should suggest an appropriate intercept path for the SAM.

Arc tangent

Arc tangent is the most accurate method. The student will need either a calculator with trigonometric functions or a table of trigonometric functions. When the computer gives the north and east coordinate of the ICBM, the students use this formula:

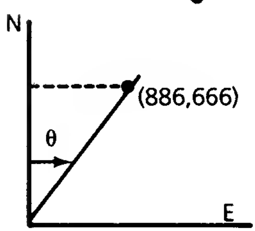
$$\text{TAN } \theta = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{\text{east}}{\text{north}}$$

They can either calculate the angle on the calculator by keying INV. TAN. or look up the TAN θ value on a table of tangents. This angle is then put in as the SAM heading.

For example,

ICBM	
N	E
886	666

$$\text{Tan } \theta = \frac{666}{886} = .751693$$

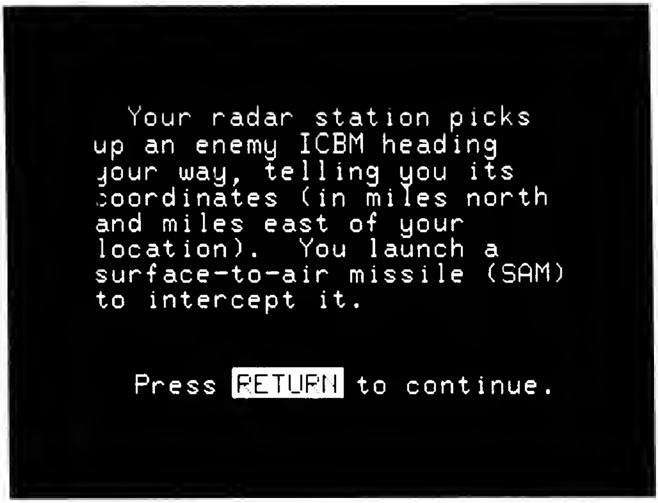


θ = ARCTAN (.751693) = .646 radians = 37°

Once the student arrives at the angle by method 3 or 4, it should be entered each time the computer requests a heading. The student may need to adjust the heading slightly when using method 3, but method 4 should be accurate and need no adjustment.

Sample Runs

ICBM simulates the interception of an ICBM by a SAM missile.



ICBM has students attempt to intercept an incoming ICBM missile by calculating the coordinates for their SAM missile that will put the SAM on an interception course. In this situation the angle, whose tangent equals (666/886) = .7517, would be 37°. The best heading is 37°.

Examples of Screen Output

Sample Runs

```
     ICBM               SAM
MILES    MILES    MILES    MILES    HEADING
NORTH    EAST     NORTH    EAST        ?
-----    -----    -----    -----    -------
 326      225       0        0         60

ICBM & SAM ARE NOW 284 MILES APART.
 276      190      29       50         20

ICBM & SAM ARE NOW 167 MILES APART.
 227      156      83       70         32

ICBM & SAM ARE NOW 50 MILES APART.
 177      122     132      100
```

Students can do this by guessing ...

```
     ICBM               SAM
MILES    MILES    MILES    MILES    HEADING
NORTH    EAST     NORTH    EAST        ?
-----    -----    -----    -----    -------
 886      666       0        0          37
ICBM & SAM ARE NOW 310 MILES APART.
 506      455      357      269         37
ICBM & SAM ARE NOW 196 MILES APART.
 566      425      408      308         37
ICBM & SAM ARE NOW 82 MILES APART.
 526      395      460      346         37
CONGRATULATIONS - YOUR SAM CAME
WITHIN 8 MILES OF THE ICBM AND
DESTROYED IT.

      Press RETURN to continue.
```

... or by doing some calculations that give them an accurate heading from the start.

Examples of Screen Output

The ATARI Learning Systems Graphics program was developed by the Minnesota Educational Computing Consortium (MECC). It was converted from existing MECC microcomputer programs in *Mathematics Volume 1*, which were adapted from many sources. Programming revisions on the ATARI computer diskette were done by Charles Erickson, MECC. Screen layout revisions were designed by Lois Edwards, MECC.

Slope was written by Marge Kosel, MECC. It was converted for the ATARI computer by Darrell Ricke, MECC. Polygraph and Polar were written by Darrell Ricke and Kevin Hausmann, MECC. They were converted for the ATARI computer by Tom Boe and Darrell Ricke, MECC. ICBM was contributed through the MECC Timeshare System Library by Peter Arcand of Mahtomedi High School, Mahtomedi, NM. The conversion to the ATARI computer was done by Ed Bertsch, MECC. Snark originated with People's Computer Company in Menlo Park, California. The conversion to the ATARI computer was done by Mike Boucher, MECC. Radar was developed and programmed by Todd Bailey, MECC. The conversion to the ATARI computer was done by Lance Allred, MECC.

This manual was compiled and revised by Lois Edwards from material found in *Mathematics Volume 1*. That material was written by Linda Borry, Kevin Hausmann, Helen Kock, and Marge Kosel, MECC, with contributions from Don Nitzkowski, St. Paul Public Schools.

Every effort has been made to ensure the accuracy of the product documentation in this manual. However, because we are constantly improving and updating our computer software and hardware, Atari, Inc. is unable to guarantee the accuracy of printed material after the date of publication and disclaims liability for changes, errors or omissions.

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